NAG Toolbox for MATLAB

f07ae

1 Purpose

f07ae solves a real system of linear equations with multiple right-hand sides,

$$AX = B \qquad \text{or} \qquad A^{\mathrm{T}}X = B,$$

where A has been factorized by f07ad.

2 Syntax

```
[b, info] = f07ae(trans, a, ipiv, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07ae is used to solve a real system of linear equations $AX = B$ or $A^{\mathrm{T}}X = B$, the function must be preceded by a call to f07ad which computes the LU factorization of A as $A = PLU$. The solution is computed by forward and backward substitution.

If **trans** = 'N', the solution is computed by solving $PLY = B$ and then $UX = Y$.

If **trans** = 'T' or 'C', the solution is computed by solving $U^{\mathrm{T}}Y = B$ and then $L^{\mathrm{T}}P^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **trans – string**

Indicates the form of the equations.

trans = 'N'

$AX = B$ is solved for X.

trans = 'T' or 'C'

$A^{\mathrm{T}}X = B$ is solved for X.

Constraint: **trans** = 'N', 'T' or 'C'.

2: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The LU factorization of A, as returned by f07ad.

3: **ipiv**($*$) **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

The pivot indices, as returned by f07ad.

4: **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a** The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **trans**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **ipiv**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon P|L||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\| |A^{-1}||A||x| \right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\| |A^{-1}||A| \right\|_\infty \leq \kappa_\infty(A)$.

Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$, and $\mathrm{cond}\left(A^{\mathrm{T}}\right)$ can be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07ah, and an estimate for $\kappa_\infty(A)$ can be obtained by calling f07ag with **norm_p** = 'I'.

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to f07ah to refine the solution and return an error estimate.

The complex analogue of this function is f07as.

9 Example

```
trans = 'N';
a = [5.25, -2.95, -0.95, -3.8;
            0.3428571428571429,   3.891428571428571,   2.375714285714285,
0.4128571428571428;
           0.300952380952381,  -0.4631179637787567,  -1.513859275575135,
0.2948206069505628;
         -0.2114285714285714,  -0.3298825256975037,  0.004723367663983707,
0.1313732394878517];
ipiv = [int32(2);
     int32(2);
     int32(3);
     int32(4)];
b = [9.52, 18.47;
     24.35, 2.25;
     0.77, -13.28;
     -6.22, -6.21];
[bOut, info] = f07ae(trans, a, ipiv, b)
```

```
bOut =
    1.0000    3.0000
   -1.0000    2.0000
    3.0000    4.0000
   -5.0000    1.0000
info =
           0
```